Hydromagnetic Mixed Convection Stagnation-Point Flow of Casson Nanofluid over a Nonlinear Stretching Sheet with Ohmic Heating

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Abstract

This study focuses on a mixed convection transport of an electrically conducting and dissipative Casson nanofluid over a nonlinear stretchable sheet in the neighbourhood of a stagnation point. The implications of porous medium, chemical reaction coupled with activation energy, thermophoresis, Ohmic heating alongside with varying non-uniform heat source and thermal conductivity are investigated. Similarity transformation technique is employed to modify the transport nonlinear partial differential equations into ordinary ones while the translated set of equations is integrated by means of shooting method accompanied with Runge-Kutta Fehlberg algorithm. The significant contributions of the embedded parameters on the flow, heat and mass transport are graphically and tabularly presented and deliberated while the numerical results strongly agree with related published studies in the limiting conditions. It is found that a lift in the strength of Casson fluid parameter decelerates the fluid flow while enhancing the viscous drag and thermal profiles. The inclusion of the nonlinear convection term aids fluid flow whereas heat transfer declines with an uplift in the thermophoresis and Brownian motion terms.

Keywords: Casson nanofluid; Ohmic heating; Mixed convection; Stagnation-point flow.

Introduction

The investigations of the non-Newtonian fluids are on the increase in the recent times due to its extensive engineering and industrial applications ranging from food processing, crude oil extraction, biomedical engineering (such as fluid flow in brains and blood flows) to pharmaceuticals. Various model of non-Newtonian fluid exists such as the micropolar fluid, Maxwell fluid, Casson fluid, etc. Casson fluid demonstrates a shear thinning characteristics. It is prominent among others owing to its distinct property of zero viscosity at an infinite rate of shear while exhibiting an infinite viscosity at zero rate of shear (Mythili & Sivaraj, 2016). The model was invented by Casson (1959) to analyze the transport characteristics of pigment-oil suspensions of printing ink with intrinsic yield stress attribute. The suitability of this model to adequately describe the rheological behaviour of various ingredients such as paints, lubricants, jelly, tomato sauce, blood, honey, etc has been reported by various authors (Mondal et al., 2018; Raju et al., 2016; Singh et al., 2018; Nadeem et al., 2012).

Conventional base fluids (water, oil, ethanol glycol, etc.) have been found to be of low thermal conductivity and thus offer low heat transfer rates. A new class of heat transfer fluid originated by Choi and Eastman (1995) is the nanofluid (Dzulkifli et al., 2018). It offers an improved thermal conductivity required in various engineering/manufacturing devices such as in pharmaceutical processes, cooling of engines/vehicles, etc. Nanofluid describes the suspension of nanometer particles in base fluids for an improved thermal conductivity in comparison to the conventional base fluids. Due to huge applications of nanofluids, several reports have been presented in literature on the subject with various assumptions and geometries (see Alsaedi et al., 2020; Hafeez et al., 2020; Abolbashari et al., 2015; Ahmad et al., 2016; Noor et al., 2015). In this study, the transport of a nonlinear mixed convection reactive Casson nanofluid is examined over a nonlinearly stretching sheet with effect of Ohmic heating.

Many engineering and material manufacturing processes such as nuclear power plants, hot rolling, electrical power generation, etc require high temperature. In such operations, the knowledge of thermal radiation plays a vital role for the construction of energy conversion devices. In cases where the magnitude of temperature difference is high within the flow, the modelling of the radiative heat flux as linear type becomes inoperative, thus, it is imperative to apply the more general nonlinear type to capture the effect of radiation. For instance, Al-
Khaled et al. (2020) investigated the impact of nonlinear thermal radiation on the transport of a reactive tangent hyperbolic fluid whereas Fatunmbi and Adeniyan (2020) reported such influence on micropolar fluid while Khan et al. (2018) carried out such investigation with an internal heat source in the neighborhood of a stagnation-point. Mixed convection transport over linearly stretchable surfaces offers immense engineering and industrial applications as found in the cooling of nuclear reactor, metallurgical and extrusion activities, glass blowing, etc. Following Crane (1970), the case of linearly stretching surface has been widely investigated by various authors (see Akinbobola & Okoya, 2015; Mabood et al., 2017; Fatunmbi & Adeniyan, 2018). However, in practical situations such as in annealing of copper wires and drawing of plastic sheet, linearity of the stretching sheet velocity is unrealistic as the sheet velocity can be exponential and nonlinear. This type was first reported by Gupta and Gupta (1977). Thereafter, various authors have extended such phenomenon (see Magyari & Keller, 1999; Cortell, 2007; Waqas et al., 2016; Fatunmbi et al., 2020).

The combination of both free and forced convection flow is referred to as mixed convection. The significant of such concept can be encountered in drying processes, cooling of fans and electronic appliances, solar power collectors, etc. Previous investigators assumed a linear density variation in the buoyancy force term but the occurrence of high temperature difference between the surface and the ambient proves such an assumption unrealistic. For accurate prediction of the flow, heat and mass transfer in the boundary layer, the incorporation of nonlinear density variation with temperature and concentration becomes non-negotiable. In this regard, the present study therefore aims to investigate the transport of a nonlinear mixed convection reactive Casson nanofluid in the neighbourhood of a stagnation point in a porous medium with nonlinear thermal radiation and activation energy.

To develop the governing equations modelling the problem under consideration, it is assumed that the flow is incompressible, viscous and steady. The working fluid is a hydromagnetic Casson nanofluid configured in a two-dimensional vertically stretchable sheet with zero mass flux at the sheet. An external magnetic field is applied perpendicular to the flow axis with non-uniform strength given as $B(x) = B_0 x^{(m-1)/2}$ while neglecting the induced magnetic field impact based on sufficiently low magnetic Reynolds number. The flow is in the neighborhood of a stagnation point in the direction of $(x)$ while $(y)$ axis is normal to it. The velocity components in the leading edge and normal directions are assumed to be $(u, v)$ respectively as indicated in Fig. 1.

![Fig. 1 The Flow Configuration](image-url)
The stretching sheet has the velocity \( u = U_a = \alpha x^m \) while the velocity upstream is indicated as \( u = U_b = \beta x^m \) where \( \alpha > 0, \beta \) and \( m \) respectively describe the stretching rate, a constant which measures the magnitude of stagnation point flow and power law exponent. Assuming also a temperature-reliant Casson nanofluid thermal conductivity and nonlinear thermal radiation in the energy equation (see Eq. 7). The implications of Ohmic and frictional heating and non-uniform heat source/sink associated with thermophoresis and Brownian motion are also incorporated in the heat transfer equation. The Casson nanofluid density \( \rho \) variations with temperature and concentration modelled in the momentum equation are taken to be nonlinear in nature and expressed as (see Mandal & Mukhopadhyay, 2018).

\[
\rho(T) = \rho(T_w) + \left( \frac{\partial \rho}{\partial T} \right) (T - T_w) + \left( \frac{\partial^2 \rho}{\partial T^2} \right) (T - T_w)^2 + ...
\]

\[
\rho(N) = \rho(N_w) + \left( \frac{\partial \rho}{\partial N} \right) (N - N_w) + \left( \frac{\partial^2 \rho}{\partial N^2} \right) (N - N_w)^2 + ...
\]

The expansion of Eqs. (1-2) up to the second order respectively gives

\[
\frac{\partial \rho}{\partial T} = -\beta_1 (T - T_w) - \beta_2 (T - T_w)^2,
\]

\[
\frac{\partial \rho}{\partial N} = -\beta_3 (N - N_w) - \beta_4 (N - N_w)^2.
\]

With respect to the principles of the boundary layer approximations coupled with the above raised assumptions, the equations listed in (5-8) describe the transport equations for the nonlinear mixed convection hydromagnetic Casson nanofluid (Prasad et al., 2011; Khan et al., 2018).

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
u \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = U_a \frac{\partial u}{\partial x} + \frac{\partial f}{\partial y} \left( 1 + \frac{1}{\gamma} \right) - \left( \frac{\partial g}{\partial x} \right) + \frac{\mu_f}{\nu_f P_f} (u - U_b) + \frac{\mu}{K_p} (u - U_b)^2 + \frac{\mu}{K_p} \left[ \left( k \frac{\partial T}{\partial y} \right) + \left( k \frac{\partial N}{\partial y} \right) + \left( k \frac{\partial \phi}{\partial y} \right) \left( \frac{\partial T}{\partial r} \right) + \left( \frac{\partial N}{\partial r} \right) \left( \frac{\partial \phi}{\partial r} \right) \right] + \frac{\mu}{K_p} \left( u - U_b \right)^2 + \frac{\mu}{K_p} \left( u - U_b \right)^2 + \frac{1}{\gamma} \left( \frac{\partial T}{\partial y} \right) + \frac{1}{\gamma} \left( \frac{\partial N}{\partial y} \right) + \frac{1}{\gamma} \left( \frac{\partial \phi}{\partial y} \right) \frac{T_m^2}{r_m^2} + \frac{1}{\gamma} \left( N_m^2 \right) + \frac{1}{\gamma} \left( \phi_m \right) \frac{T_m^2}{r_m^2}.
\]

\[
\frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = D_B \frac{\partial N^2}{\partial y^2} + \frac{D_T}{\partial r_m} \frac{\partial N}{\partial y} + \frac{\partial \phi}{\partial r_m} \frac{T_m}{r_m^2}.
\]

The model for non-uniform thermal conductivity is expressed as (see Rahman et al., 2010).

\[
k = \frac{k_{c}}{r_{m} \omega} \left[ (T_w - T_m) + \xi (T - T_m) \right]
\]

More so, the non-uniform heat source \( (P''') \) appearing in the last term of Eq. (7) is modelled as

\[
k = \frac{k_{c}}{r_{m} \omega} \left( A f' + A' \theta (T_w - T_m) \right)
\]

The accompanied boundary conditions for equations (5-8) are:

\[
u = U_a = \alpha x^m, v = 0, T = T_w = T_m + B x^3, N = N_w \text{ when } y = 0,
\]

\[
u = U_b = \beta x^m, T = T_m, N = N_w \text{ with } y \to \infty.
\]

The expression \( h^2 \left( N - N_w \right) \left( \frac{T}{T_m} \right)^{\alpha} \exp \left( -\frac{E_a}{R_T} \right) \) presented in Eq. (8) typifies the modified Arrhenius term. Here \( h^2 = \frac{h_o \omega}{x^{m-1}} \) implies the chemical rate \( h_o \) is a constant. Similarly, \( A / A' \) corresponds to space/temperature dependent heat source/sink, \( K_m \) describes the constant thermal conductivity, \( \xi \) relates to its parameter, \( K_P = K_2 x^{(1-m)} \) symbolizes permeability of the porous medium with \( K_2 \) being a constant.

The underlisted dimensionless variables are introduced into the transport Eqs. to translate them into ordinary differential Eqs.
Incorporating the dimensionless variables \((12)\) in the governing Eqs. (5-8) with the implications of Eqs. (9-10) results to the underlisted:

\[
\left(1 + \frac{1}{\gamma}\right) f'''' + \frac{2m}{m+1} (f'' - B^2) - \frac{2mH(\delta \theta)}{m+1} \left(\frac{1}{f'} - B\right) + \lambda_2 \delta_2 \phi(1 + \delta_2 \phi)
\]  

\[
\frac{1}{Pr} \left[ \frac{\gamma}{\gamma} \left[ 1 + \left[ \frac{\gamma}{\gamma} + \frac{1}{\gamma} \right] \frac{\theta''''}{\gamma} + \frac{1}{\gamma} \left( \frac{\gamma}{\gamma} + \frac{1}{\gamma} \right) \phi'' + \frac{1}{\gamma} \left( \frac{\gamma}{\gamma} + \frac{1}{\gamma} \right) \delta_2 \frac{\phi'''}{\gamma} \right] + \frac{1}{\gamma} \left( \frac{\gamma}{\gamma} + \frac{1}{\gamma} \right) \phi'' + \frac{1}{\gamma} \left( \frac{\gamma}{\gamma} + \frac{1}{\gamma} \right) \delta_2 \frac{\phi'''}{\gamma} \right] \right)
\]  

while the wall conditions translates to:

\[
\begin{align*}
(f'(0) &= 1, f(0) = 0, 0.0(0) = 1, \phi(0) = 1, \\
(f'(\infty) &= B, \theta(\infty) = 0, \phi(\infty) = 0.
\end{align*}
\]

Table 1 below depicts the nomenclature of the symbols used in this study.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(B)</td>
<td>Stretching parameter</td>
<td>(\delta_1)</td>
<td>Nonlinear thermal convection</td>
</tr>
<tr>
<td>(Re)</td>
<td>Reynolds number</td>
<td>(\beta_1)</td>
<td>Coefficient of linear thermal expansion</td>
</tr>
<tr>
<td>(Ec)</td>
<td>Eckert number</td>
<td>(\beta_2)</td>
<td>Coefficient of nonlinear thermal expansion</td>
</tr>
<tr>
<td>(g)</td>
<td>Gravitational acceleration</td>
<td>(A^*)</td>
<td>Coefficient temperature-dependent heat source</td>
</tr>
<tr>
<td>(H)</td>
<td>Magnetic field parameter</td>
<td>(\delta_2)</td>
<td>Nonlinear mass convection</td>
</tr>
<tr>
<td>(E)</td>
<td>Activation Energy</td>
<td>(n)</td>
<td>Temperature exponent</td>
</tr>
<tr>
<td>(Da)</td>
<td>Darcy parameter</td>
<td>(\beta_3)</td>
<td>Coefficient of linear solutal expansion</td>
</tr>
<tr>
<td>(u, v)</td>
<td>Velocity in (x, y) direction</td>
<td>(\beta_4)</td>
<td>Coefficient of nonlinear solutal expansion</td>
</tr>
<tr>
<td>(NT)</td>
<td>Thermophoresis parameter</td>
<td>(N)</td>
<td>Brownian motion parameter</td>
</tr>
<tr>
<td>(Pr)</td>
<td>Prandtl number</td>
<td>(\lambda_1)</td>
<td>Mixed convection parameter</td>
</tr>
<tr>
<td>(Rd)</td>
<td>Radiation parameter</td>
<td>(\theta_b)</td>
<td>Temperature ratio</td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature</td>
<td>(\kappa)</td>
<td>Ratio of concentration to buoyancy forces</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Kinematic viscosity</td>
<td>(N)</td>
<td>Concentration of nanoparticles</td>
</tr>
<tr>
<td>(Gr)</td>
<td>Local Grashof number</td>
<td>(Gn_k)</td>
<td>Local solutal Grashof number</td>
</tr>
</tbody>
</table>
Furthermore, the expressions for the coefficient of skin friction ($C_{f_2}$) as well as the Nusselt number ($Nu_x$) and the Sherwood number $Sh_x$ are respectively presented in Eq. (17). These expressions are the physical quantities of engineering concern for this study.

$$C_{f_2} = 2\tau_w (\frac{\rho \nu}{\mu})^{-1}, Nu_x = xq_w[k_m(\tau_w - T_w)]^{-1}, Sh_x = xq_w[D_B(N_w - N_m)]^{-1}. \quad (17)$$

with

$$\tau_w = \mu \left(1 + \frac{1}{\gamma}\right)(\frac{\partial u}{\partial y})_{y=0}, \quad q_w = -\left(\frac{16\gamma T^2}{2\sigma} - \frac{\partial T}{\partial y}\right)_{y=0}, \quad q_m = -\left(D_B \frac{\partial N}{\partial y}\right)_{y=0}. \quad (18)$$

here $\tau_w$ corresponds to surface shear stress whereas $q_w$ ($q_m$) typifies surface heat (mass) flux in that order. With the substitution of Eqs. (12) and (18) into (17), the quantities in Eq. (17) respectively results to the following the dimensionless terms as presented in Eq. (19).

$$\bar{C}_{f_2} = f''(0), \bar{Nu}_x = -\theta'(0), \bar{Sh}_x = -\phi'(0). \quad (19)$$

Where

$$\bar{C}_{f_2} = \frac{1}{2(\alpha + 2\beta(1 - \alpha))} \bar{Nu}_x = \frac{1}{[\frac{\alpha}{2}]^3 \bar{Sh}_x = \frac{1}{[\frac{\alpha}{2}]^3}. \quad (20)$$

### Numerical Method with validation

Due to the high nonlinearity nature of the boundary value problem (13-16), a numerical technique via shooting method in company with Runge-Kutta-Fehlberg algorithm has been employed for the solutions. Notable authors have applied and described in details the effectiveness of this technique. For detail explanation of this technique (see Fatunmbi & Adeniyan, 2020; Xu & Lee, 2013; Attili and Syam, 2008; Mahanthesh et al., 2018). For the computations, the following values have been carefully selected as default parametric values $B = 0.2, H = K = e, \xi = 0.2, Rd = 0.3, E\xi = 0.1, \delta_1 = \delta_2 = \kappa = 0.5 = \lambda_1 = NT = N\beta, Pr = 2.0, \theta_0 = 1.5, A = A^* = 0.1 = D\alpha, S_c = 0.44, E = y = 0.3, n = 1.0$ except if stated otherwise in the various graphs. The code for the solutions developed in this study have been verified by comparing the computational values of some chosen parameters with related published works in literature for limiting scenarios. Table 2 gives the record of the Nusselt number $Nu_x$ as compared with Grubka and Bobba (1985) for changes in temperature exponent term $n$ and Prandtl number $Pr$. The comparison shows a good relationship as depicted in that table. More so, the variations in the nonlinear stretching term $m$ with respect to the skin friction coefficient $C_{f_2}$ are compared with Makinde (2010) and Cortell (2007) in the limiting
conditions. A strong relationship exists in the obtained results with those of the given authors as typified in Table 3. These comparisons offer a confirmation to the validity of the current numerical solutions.

**Table 2:** Computational values of $\bar{N}u_x$ with respect to variations in $n$ and $Pr$ as compared with published data

<table>
<thead>
<tr>
<th>$n$</th>
<th>$Pr = 1.0$</th>
<th>$Pr = 10.0$</th>
<th>$Pr = 100.0$</th>
<th>$Pr = 1.0$</th>
<th>$Pr = 10.0$</th>
<th>$Pr = 100.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.0</td>
<td>-1.0000</td>
<td>-10.0000</td>
<td>-100.0000</td>
<td>-1.0000</td>
<td>-10.0000</td>
<td>-100.0000</td>
</tr>
<tr>
<td>-1.0</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.00012</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>0.0</td>
<td>0.5820</td>
<td>2.3080</td>
<td>7.7657</td>
<td>0.58201</td>
<td>2.30800</td>
<td>7.76565</td>
</tr>
<tr>
<td>1.0</td>
<td>1.0000</td>
<td>3.7207</td>
<td>12.2940</td>
<td>1.00000</td>
<td>3.72067</td>
<td>12.29408</td>
</tr>
<tr>
<td>2.0</td>
<td>1.3333</td>
<td>4.7969</td>
<td>15.7120</td>
<td>1.33333</td>
<td>4.79687</td>
<td>15.71197</td>
</tr>
<tr>
<td>3.0</td>
<td>1.61534</td>
<td>5.6934</td>
<td>18.5516</td>
<td>1.61538</td>
<td>5.69338</td>
<td>18.55154</td>
</tr>
</tbody>
</table>

**Table 3:** Variations in $\bar{m}$ when other parameters are zero with respect to $\bar{C}_f$ as compared with published data

<table>
<thead>
<tr>
<th>$\bar{m}$</th>
<th>Cortell (2007)</th>
<th>Makinde (2010)</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.627547</td>
<td>0.627600</td>
<td>0.627563</td>
</tr>
<tr>
<td>0.2</td>
<td>0.766758</td>
<td>0.766900</td>
<td>0.766945</td>
</tr>
<tr>
<td>0.5</td>
<td>0.889477</td>
<td>0.889500</td>
<td>0.889552</td>
</tr>
<tr>
<td>1.0</td>
<td>1.000000</td>
<td>1.000000</td>
<td>1.000008</td>
</tr>
<tr>
<td>3.0</td>
<td>1.148588</td>
<td>1.148600</td>
<td>1.148601</td>
</tr>
<tr>
<td>10.0</td>
<td>1.234875</td>
<td>1.234900</td>
<td>1.234882</td>
</tr>
<tr>
<td>100.0</td>
<td>1.276768</td>
<td>1.276800</td>
<td>1.276781</td>
</tr>
</tbody>
</table>

**Results**

This section analyzes graphically the significant contributions of the main parameters on the various dimensionless quantities (velocity $f'(\eta)$, temperature $\theta(\eta)$, concentration $\phi(\eta)$, skin friction coefficient $\bar{C}_f$, Nusselt number $\bar{N}u_x$ and Sherwood number $\bar{S}h_w$). These contributions are presented in Figures 2-23 with appropriate discussions.

The implication of varying the magnetic field term $H$ on the profile of velocity is described in Fig. 2 in the presence of Casson fluid term $\gamma$. The figure informs the shrinking nature of the hydrodynamic boundary structure with a hike in $H$ as well as $\gamma$. 
This decelerated flow due to $H$ as observed in Fig. 2 is occasioned due to Lorentz force produced by the interaction of the applied magnetic field and the electrically conducting Casson nanofluid. Likewise, the reduction in the fluid flow owing to a rise in $\gamma$ indicates that growth in $\gamma$ compels a reduction in the transport field due to a fall in the yield stress as $\gamma$ increases which in turn lowers the fluid flow.

Besides, a rise in $\gamma$ strengthens the plastic dynamic viscosity above the Casson fluid viscosity and at such, the flow is resisted as further depicted in Fig. 3. It is to be remarked that the non-Newtonian attributes vanish when $\gamma \to \infty$ and at such, the fluid purely exhibits Newtonian fluid property. Also, in relation to the fluid flow, the velocity profile depletes with respect to higher values of the nonlinear stretching term $m$. For the linearly stretching scenario, the velocity field is higher than that of nonlinearly stretching case as depicted in figure 3.
The plot depicting the velocity field versus $\eta$ for varying mixed convection term $\lambda_1$ in the existence of the velocity ratio term $B$ is sketched in Fig. 4. The impact of $\lambda_1$ is to boost the velocity field owing to a decline in the viscous force as $\lambda_1$ increases. Also, the velocity field heightens when $B$ is raised as noticed in Fig. 4. This accelerated flow occurs due to the fact that the upstream velocity is higher than that of the wall velocity.

From Fig. 5, it is observed that the transport field accelerates when the ratio of the concentration to buoyancy forces ($\chi$) increases while the converse is the case for a rise in the Darcy term ($Da$). The thermal nonlinear mixed convection parameter ($\delta_1$) boosts the velocity profile as showcased in Fig. 6. Here, a hike in $\delta_1$ leads to a rise in $(T_\infty - T_w)$ and at such, the velocity field accelerates. The nanoparticles concentration field escalates with higher values of the thermophoresis parameter $NT$ as demonstrated in Fig. 7. On the contrary, the concentration field decays with a hike in Brownian motion parameter $Nb$. Furthermore, growth in the chemical reaction term $K$ and the Schmidt number $Sc$ shrink the solutal boundary layer structure which in turn dictates a reduction in the concentration profile as found in Fig. 8.
In this figure, an improvement in $K$ describes a hike in the rate of destructive chemical reaction through in which the species dissolve more effectively and as a result, the nanomaterial concentration profile depreciates.

The thermal boundary layer structure expands with rising values of the thermophoresis term $NT$ as indicated in Fig. 9. This can be attributed to the the a rise in the temperature gradient as $NT$ increases. A raise in the magnitude of the Brownian motion $N_b$ also strengthens the temperature profile. The Brownian motion describes an irregular movement exhibited by the nanoparticles suspended in a base fluid. In respect to this irregular motion, there is a higher kinetic energy owing to the enhanced movement of the molecules of both the nanoparticles as well as the base fluid which leads to an improved surface temperature. The reaction of the space-dependent heat source parameter $A$ with respect to temperature distribution in the existence/non-existence of the radiation term $Rd$ is sketched in Fig. 10. Advancing the values of $A$ causes an improvement in the surface temperature whether $Rd$ is present or absent. However, in the absence of $Rd$, the temperature is lower than in its presence. Obviously, the thermal boundary structure is energized with a rise in $Rd$ and consequently propel a boost in the temperature profile as depicted in Fig. 10. The implication of the temperature-dependent heat source $A^*$ on the thermal field is plotted in Fig. 11 in the existence or otherwise of the Eckert number $Ec$. In the presence of $A^*$, an additional heat is created which is responsible for the hike in temperature profile. Moreover, the inclusion of $Ec$, the thermal field also escalates due to friction between the fluid particles. Eckert number corresponds to the ratio of flow kinetic energy to that of the boundary layer enthalpy difference. In this regard, a raise in $Ec$ enhances the production of heat and at such, compels an a rise in the temperature field. The impact
of the temperature ratio term $\theta_b$ is to improve the temperature profile as clearly described in Fig. 12. The temperature parameter $\theta_b$ corresponds to the ratio of the sheet temperature to that of the upstream temperature, i.e. $\theta_b = \frac{T_s}{T_b}$ Hence, a rise in $\theta_b$ corresponds to higher temperature at the stretching sheet and at such, a rise in the surface temperature. The graph of temperature versus $\eta$ for variations in the Casson fluid parameter $\gamma$ for linear ($m = 1$) and nonlinear stretching sheet ($m \neq 1$) is captured in Fig. 13.

![Graph of temperature versus $\eta$](image1)

**Fig. 14** Temperature field for changes in $m$

**Fig. 15** Sherwood profile for changes in $K$ & $Sc$

There is an improvement in the temperature distribution with higher values of $\gamma$ for both linear/nonlinear stretching sheet. However, higher surface temperature occurs with nonlinear stretching sheet as found in Fig. 13. Figure 14 informs about the reaction of temperature profile to the changes in the nonlinear stretching term $m$ for variations in the wall temperature exponent parameter $n$. The thermal boundary structure expands with $m$ in the existence of higher $n$ and thus, temperature distribution heightens. The trend is however reversed with lower $n$ as temperature field depreciates with variations in $m$. The nature of the mass transfer ($\tilde{S}_h$) with respect to changes in the activation energy $E$ and for variations of chemical reaction term $K$ and Schmidt number $Sc$ is demonstrated in Fig. 15. Clearly, the presence of $K$ and $Sc$ aid mass transfer ($\tilde{S}_h$) whereas for any fixed $K$ and $Sc$, the an increase in $E$ lowers $\tilde{S}_h$. This pattern informs that a rise in $E$ causes the modified Arrhenius function to decline. Thus, there is a lift in the productive chemical reaction and at such, the concentration field becomes enlarged and consequently lowers $\tilde{S}_h$.

![Graph of mass transfer versus $E$](image2)

**Fig. 16** Variations of $\gamma$ & $m$ on $\bar{C}_{fx}$

**Fig. 17** Variations of $Da$ & $B$ on $\bar{C}_{fx}$
The drag force ($\vec{C}_{fx}$) is strengthened with higher values of the Casson fluid term $\gamma$ and the nonlinear stretching term $m$ as illustrated in the Fig. 16. However, for fixed values of $\gamma$ and $m$, an uplift in the nonlinear convection parameter $\delta_c$ peters out $\vec{C}_{fx}$. Likewise, a hike in the Darcy term $Da$ raises $\vec{C}_{fx}$ in the existence or otherwise of the velocity ratio parameter $B$ as depicted in Fig. 17. Meanwhile, the inclusion of $B$ decreases the skin friction coefficient as noticed in this figure. The inclusion of the wall temperature term $\eta$ on the thermal field aids the heat transfer mechanism ($\vec{Nu}_x$) whereas with higher $Nb$ and $NT$, $\vec{Nu}_x$ declines as demonstrated in Fig. 18. Similarly, $\vec{Nu}_x$ depreciates with higher values of radiation term $\vec{Rd}$ and the thermal conductivity parameter $\xi$ as displayed in Fig. 19.

![Fig. 18 Effects of $NT$&$Nb$ on $\vec{Nu}_x$](image1.png)  ![Fig. 19 Impact of $NT$&$Nb$ on $\vec{Nu}_x$](image2.png)

**Conclusion**

A numerical analysis has been performed on the transport of a quadratic mixed convection hydromagnetic reactive Casson nanofluid in the neighbourhood of a stagnation point. The flow is configured in a two-dimensional nonlinear vertically stretchable sheet enclosed in a porous medium with the impact of nonlinear thermal radiation coupled with activation energy. The nonlinear boundary value problem has been solved by integrated with the Runge-Kutta Fehlberg coupling shooting technique. The solutions are presented graphically and deliberated while comparisons with earlier studies show good agreement in the limiting situations. The study reveals that:

- An uplift in the Casson parameter $\gamma$ strengthens the skin friction coefficient and thermal field while it shrinks the hydrodynamic boundary structure and the fluid flow decelerates.

- Growth in the velocity ratio parameter $B$ upsurges fluid velocity whereas the viscous drag effect declines while the converse is true for the nonlinear stretching term $m$.

- The fluid flow decelerates with rising values of the magnetic field term $H$ and Darcy parameter $Da$ whereas there is an improvement in the Casson nanofluid temperature with the increased values of the radiation $\vec{Rd}$, temperature ratio term $\theta_b$, thermophoresis $NT$ and Eckert number $Ec$ parameters.

- Mass transfer improves with higher values of Schmidt number $Sc$ and chemical reaction parameters. Also, the concentration profile advances with thermophoresis term $NT$ while the reverse is the case for the Brownian motion $Nb$.

- Skin friction coefficient is strengthened due to higher magnetic field term $H$, power law exponent $\kappa$ and Darcy parameters $Da$ while a reverse trend is found with a rise in $H$.

- Nusselt number appreciates in respect of the wall temperature exponent term $\eta$ whereas it diminishes with thermophoresis $NT$ and Brownian motion parameters $Nb$. 

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References


